PROBABILITY

Part I

How likely something is to happen

Probability of an event happening = $\dfrac{\text{Number of ways it can happen}}{\text{Total number of outcomes}}$

If there are 'n' equally likely events out of these 'm' are in the favour of 'A' then the probability of event A is $P(A) = \dfrac{m}{n}$

There are 'n–m' occasions when event 'A' does not happen. The event A does not happen is denoted by $\bar{A}$ and probability of event A does not happen is

$$P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - P(A)$$

PROBABILITY LINE

← More unlikely $0 \leq P(A) \leq 1$ More likely →

Impossible | Even chance | Certain

Rolling a 14
P = 0

Heads P = 1/2

The sun will rise
P = 1

TERMINOLOGY

	An action or operation resulting in two or more outcomes.
Sample space	A set S that consists of all possible outcomes of a random experiment.
Event	An event is defined occurrence or situation Eg : tossing a coin and the coin landing up head.
Complement of an event	The set of all out comes which are in S but not in A. The compliment event A is denoted by A, A^c, A' or not A.
Compound event	The combination of events A and B is known as Compound Event. It is denoted by AB or A and B or A ∩ B.
Mutually exclusive events	Mutually Exclusive events mean we can't get both events at the same time. It is either one or the other, but not both.
Equally likely events	When each event is as likely to occur as any other event.
Exhaustive events	A set of events is called exhaustive if all the events together consume the entire sample space.

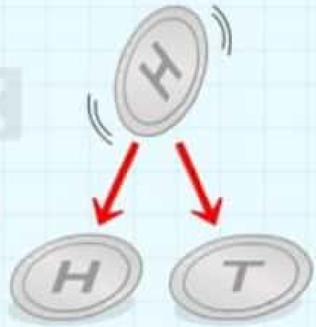
INDEPENDENT EVENTS

Independent Events are **not affected** by previous events.

This is an important concept !

Example : A coin does not know that it landed up heads before........

.... each toss of a coin is a perfect isolated thing.

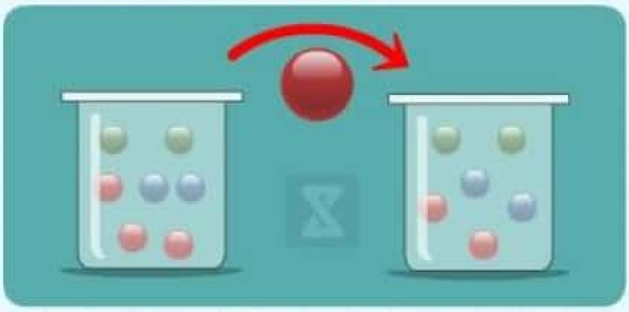


DEPENDENT EVENTS

Events that depend on what happend before

Example :

Taking colored marbles from a bag : as you take each marble, there are less marbles left in the bag, so the probabilities change.



BINOMIAL PROBABILITY DISTRIBUTION

Let probability of success of an event be p & probability of failure be q = 1 – p

The probability that the event will happen exactly 'x' times in 'n' trials is given by the probability function.

$$f(x) = P(X = x) = \binom{n}{x} p^x q^{n-x} = \frac{n!}{x!\,(n-x)!}\, p^x q^{n-x}$$

BAYE'S THEOREM

If an event A can occur only with one of the n mutually exclusive and exhaustive events $B_1, B_2, \ldots B_n$ & if the conditional probabilities of the events.

$P(A/B_1), P(A/B_2) \ldots P(A/B_n)$ are known then,

$$P(B_1/A) = \frac{P(B_1).\,P(A/B_1)}{\sum_{i=1}^{n} P(B_i).\,P(A/B_i)}$$

ADDITION THEOREM

1. If A & B are mutually exclusive events, then the probability of event A or B occuring is

$$P(A \text{ or } B) = P(A) + P(B)$$

2. If A & B are not mutually exclusive events, then

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$= P(A) + P(B \cap \overline{A}) = P(B) + P(A \cap \overline{B})$$
$$= P(A \cap B) + P(A \cap B) = P(B) + P(B \cap \overline{A})$$
$$= 1 - P(\overline{A} \cap \overline{B}) = 1 - P(\overline{A \cup B})$$

3. If A & B are mutually exclusive then $P(A \cup B) = P(A) + P(B)$

MULTIPLICATION THEOREM

DEPENDENT EVENTS OR CONTIGENT EVENTS

How to handle Dependent Events ?

$P(B \mid A)$ is called the **conditional probability** of event B given that event A has already occured.

$$P(B \mid A) = \frac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = P(A)\, P(B \mid A)$$

Note: For any three events A_1, A_2, A_3

$$P(A_1 \cap A_2 \cap A_3) = P(A_1)\, P(A_2 \mid A_1)\, P(A_3 \mid (A_1 \cap A_2))$$

INDEPENDENT EVENTS

- For two independent events A & B $P(A \cap B) = P(A) . P(B)$
- Three events A,B & C are independent if & only if all the following conditions hold:

$$P(A \cap B) = P(A) . P(B) \; ; \; P(B \cap C) = P(B) . P(C)$$
$$P(C \cap A) = P(C) . P(A) \;\&\; P(A \cap B \cap C) = P(A). P(B). P(C)$$

i.e., they must be pairwise as well as mutually independent.

- For n events A_1, A_2, A_3 .... A_n to be independent, the number of above conditions is equal to ${}^nC_2 + {}^nC_3 +$ .... ${}^nC_n = 2^n - n - 1$

Note: Independent events are not in general mutually exclusive & vice versa.